On discontinuity-induced bifurcations in a piecewise-linear capsule system

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Abstract. This work aims to investigate the occurrence conditions of discontinuity-induced bifurcations for a piecewise-linear vibro-impact capsule system. Numerical and experimental studies showed that the non-smoothness in the vibro-impact mechanism and dry friction resulted in rich and complex nonlinear dynamics [1, 2], e.g. grazing and sliding bifurcations. However, the occurrence conditions for these bifurcations are not fully understood, and this study will focus on detecting such conditions in an analytical manner for some special cases and by continuation platform COCO [4] for multi-parameter cases. Key results will be used for controlling the drift of the vibro-impact capsule system.

Introduction

Operation of a vibro-impact capsule system involving discontinuous dry friction and impact to achieve rectilinear motion is a challenging research task [1]. As shown in Figure 1(a), a capsule M_c contains an inner mass M_m via a supporting spring k_1 and a damper c. The inner mass is externally excited by a harmonic force $P_\mathrm{d}\cos(\omega t)$. Impact events occur when the relative displacement between the capsule and the inner mass exceeds the gap G, and the capsule may drift forward or backward as a whole when their interaction force overcomes the dry friction P_f. Such a vibro-impact mechanism can magnify the interaction force and drive the capsule drift even when $P_\mathrm{d} < P_\mathrm{f}$ [3]. In addition, the capsule's motion can be divided into the following six modes: *no-contact-stationary*, *no-contact-forward-drift*, *no-contact-backward-drift*, *contact-stationary*, *contact-forward-drift* and *contact-backward-drift* modes. As expected, the non-smooth impact and dry friction induce grazing and sliding bifurcations, respectively. This study aims to investigate the conditions of the occurrence of these bifurcations.

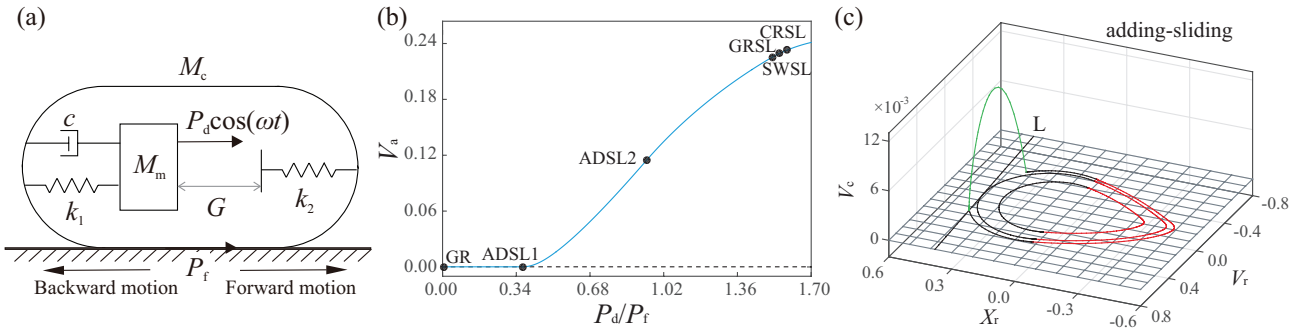

Figure 1: (a) Schematic diagram of the vibro-impact capsule system, (b) grazing and sliding bifurcations detected for various excitation amplitude, and (c) three-dimensional orbits with the amplitudes of the driven force close to the occurrence conditions of the adding-sliding bifurcation marked ADSL1.

Results and Discussion

By varying the non-dimensional excitation amplitude $P_\mathrm{d}/P_\mathrm{f}$, the non-dimensional average velocity of the capsule V_a is shown in Figure 1(b). As the amplitude increases, grazing, adding-sliding, switching-sliding, grazing-sliding and crossing-sliding bifurcations were detected in sequence, of which the adding-sliding bifurcation marked by ADSL1 resulted in forward drift and positive capsule velocity. A three-dimensional plot showing the orbits with excitation amplitudes around the value of ADSL1 is shown in Figure 1(c), indicating that the occurrence of ADSL1 brought a *contact-forward-drift* segment in the orbit. Similarly, the adding-sliding bifurcation marked by ADSL2 unlocked the *no-contact-forward-drift* mode. Occurrence conditions of these bifurcations will be investigated in this study in an analytical and numerical manner by path-following analysis [5]. Key results can be used as guidelines to optimise the design of the capsule system or to control the motion of the capsule system by adjusting the excitation amplitude and frequency.

References

[1] Guo, B., Liu, Y. (2019) Three-dimensional map for a piecewise-linear capsule system with bidirectional drifts. *Physica D* **399**:95-107.
[2] Guo, B., Liu, Y., Birler, R., Prasad, S. (2020) Self-propelled capsule endoscopy for small-bowel examination: Proof-of-concept and model verification. *Int. J. Mech. Sci.* **174**:105506.
[3] Guo, B., Ley, E., Tian, J., Zhang, J., Liu, Y., Prasad, S. (2020) Experimental and numerical studies of intestinal frictions for propulsive force optimisation of a vibro-impact capsule syste. *Nonlinear Dyn.* **101**:65-83.
[4] Dankowicz, H., Schilder, F. (2013) Recipes for continuation. SIAM.
[5] Páez Chávez, J., Liu, Y., Pavlovskaia, E., Wiercigroch, M. (2016) Path-following analysis of the dynamical response of a piecewise-linear capsule system. *Comm. Nonlinear Sci. Numer. Simulat.* **37**:102-114.