Introducing an open-source software for computing the basin stability of multi-stable dynamical systems

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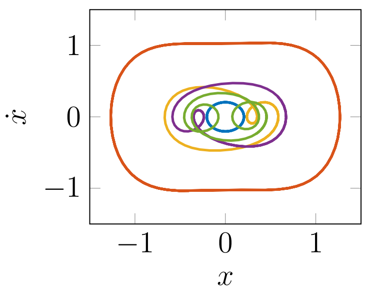
Abstract. The concept of basin stability is a recently proposed [1] global stability metric for deterministic and multi-stable systems. The computation involves extensive Monte Carlo simulations, attractor characterization, and trajectory classification. We introduce `bSTAB`, which is an open-source software project that aims at helping researchers to efficiently compute the basin stability of their custom dynamical systems.

Introduction

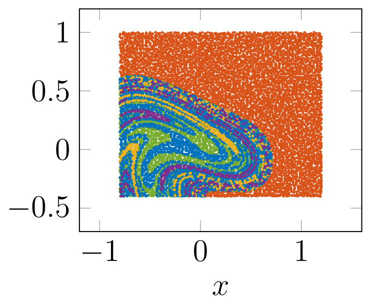
Stability analysis is one of the core exercises in nonlinear dynamics. Following the ideas of Lyapunov, most of the commonly used stability concepts rely on *local* and *small* perturbations to assess the stability of a state. Lyapunov exponents express the local stability of fixed point solutions and periodic orbits, thus indicating the capacity of the dynamical system to return to the state after a finite (but small) perturbation. Multi-stable systems exhibit multiple co-existing stable attractors, such that the asymptotic behavior solely depends on the initial state of the system. Hence, a perturbation of a state may lead the system to switch to another long-term behavior on a different attractor. The perturbation that is permissive, i.e. quantifying what *small* actually is, depends on the basins of attraction, that is the state space subset containing all initial conditions that approach the same attractor in the long term. To obtain a consistent stability picture for multi-stable systems, one needs to take the basins into account, i.e. study non-local perturbations. However, computing the basin boundaries quickly becomes challenging for high-dimensional nonlinear systems.

Results and discussion

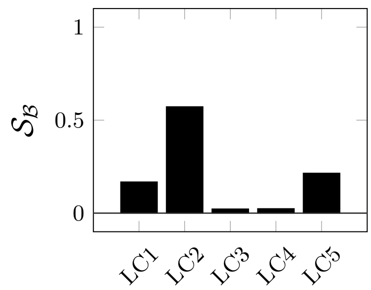
Recently, Menck et al. [1] proposed a new stability concept denoted as *basin stability* (BS) which is a probabilistic approach to estimating the basins' volumes instead of their shape. The BS gives the probability of the system to return to a state in the face of a non-small random perturbation. Hence, this approach is a global stability concept that is especially useful for high-dimensional. The basin stability has been studied extensively for networks, climate system models and others. However, the application in nonlinear (mechanical) systems seems to be in its infancy. To foster the research in those systems, we introduce an open-source software package `bSTAB` [2] that enables researchers to quickly adapt the BS concept to their systems and needs. `bSTAB` is a Matlab-based framework for computing the BS of multi-stable continuous systems. It strongly adapts the typical notion of ODE definitions in Matlab that researchers are used to. The user defines the ODE system and its parameters to study. Given the number of random samples to be drawn from a subset of the state space, i.e. a range of typically encountered initial conditions and perturbations, the program performs Monte Carlo simulations is a computationally efficient manner. Trajectories are classified to belong to one of the attractors, and the BS values are returned to the user. Parameter studies, e.g. the stability value along a model bifurcation parameter, are possible as well. We illustrate the application of `bSTAB` to a range of canonical systems from different scientific disciplines for validating our implementation.



(a)



(b)



(c)

Figure 1: The multi-stable Duffing oscillator $\ddot{x} + 0.08\dot{x} + 1x^3 = 0.2\cos(t)$ exhibiting five locally stable co-existing periodic orbits (a), the sampling of a reference set of the state space (b) and the resulting basin stability (c) of each periodic orbit LC1 (blue, period−1), LC2 (orange, period−1), LC3 (yellow, period−2), LC4 (purple, period−2), LC5 (green, period−3)

References

[1] Menck, P.J., Heitzig, J., Marwan, N., Kurths, J. (2013) How basin stability complements the linear-stability paradigm. *Nature Phys* **9** **(2)**: 89–92

[2] Stender, M., Hoffmann, N. (2020) `bSTAB`. DOI:10.5281/ZENODO.3935989. *https://github.com/TUHH-DYN/bSTAB/tree/v1.0*