Parametric Instability and Bifurcation of Thin-Walled Axially Compressed Long FRP Columns

Julio C. Coaquira*, Daniel C.T. Cardoso*, Paulo B. Gonçalves* and **Diego Orlando****

*Department of Civil and Environmental Engineering, Catholic University, Rio de Janeiro, Brazil

**Department of Mechanics and Energy, State University of Rio de Janeiro, Resende, Brazil

Abstract. There is a growing interest in the application of thin-walled beam of composite materials in several engineering fields. In the present work the nonlinear oscillations and parametric instability of a long simply supported column with a channel section made of fiber reinforced polymer is investigated. The nonlinear equations of motion of the thin-walled column are here derived in terms of the two flexural displacements and the torsion angle, taking into account large displacements, warping and shortening effects. The governing nonlinear equations of motion are discretized in space using the Galerkin method and solved by the Runge-Kutta method. The bifurcation diagrams are obtained, thus permitting the identification of the parametric instability boundaries in force control space. The parametric analysis shows how the material characteristics and geometric properties of the column influence its nonlinear vibrations and dynamic instability.

Introduction

Fiber Reinforced Polymer (FRP) profiles obtained by pultrusion techniques has been increasingly used in many structures, not only as secondary elements but also load carrying members [1]. In the pultruded columns most of the fibers are oriented along the axial direction of the element leading to higher strength and elastic constants in this direction than those in the orthogonal direction. Usually, the failure mechanism is conditioned by buckling. Here the nonlinear oscillations and parametric instability of a harmonically excited long FRP simply-supported column with a channel section with dimensions $b_f = 10cm$, $b_w = 20cm$, $t_f = 5mm$ and $t_w = 5mm$ is adopted (Fig. 1(a)). This geometry has a low torsional stiffness and flexural-torsional coupling is one of the main concern is their design. The nonlinear equations of motion of the thin-walled column are here derived in terms of the two flexural displacements and the torsion angle [2]. The longitudinal, transversal Young's modulus and shear modulus of the FRP column are given respectively by $E_1 = 15.89\text{kN/mm}^2$, $E_2 = 7.75\text{kN/mm}^2$ and $G_{12} = 3.13\text{kN/mm}^2$ while Poisson rations are $\upsilon_{12} = 0.32$ and, $\upsilon_{21} = 0.156$. The mass density and the damping ratio are $\rho = 1850\ \text{kg/m}^3$ and $\xi = 1.45\%$.

Results and Discussion

Considering a beam with length $\text{L} = 2.5m$, the lowest flexural and flexural-torsional frequencies are given respectively by $\omega_{o2} = 145.768 rad/s$ and $\omega_{o1} = 119.866 rad/s$. Figure 1(b) shows the bifurcation diagrams for $\delta = 2.0$, $\delta = 2.17$ and $\delta = 2.43$, where $\delta = \Omega/\omega_{o1}$ and Ω is the excitation frequency. The trivial solution loses stability through a supercritical flip bifurcation, giving rise to a solution of period 2 up to the escape load. Figure 1(c) shows the parametric instability and escape boundaries in the force control space (forcing magnitude vs. forcing frequency). When subjected to an axial harmonic force and, due to parametric resonance, the FRP column can lose stability at load levels much lower than the static critical load ($Q_d = 1.0$), particularly in the main parametric resonance region around two times the lowest natural frequency $\Omega = 2\omega_o$ and the resonant region around $\Omega = \omega_o$.

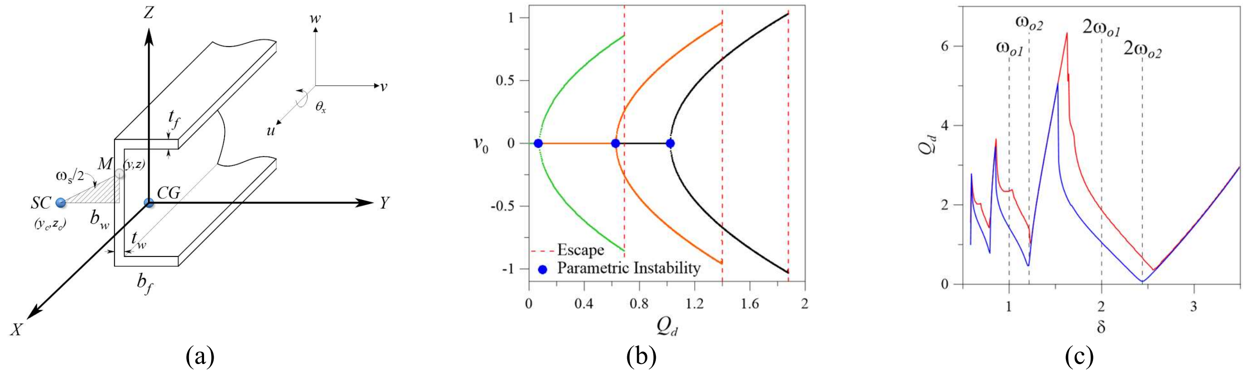

Figure 1: (a) Cross-section geometry, displacements and coordinate system; (b) Bifurcation diagrams of the Poincaré map. Black - $\delta = 2.0$ ($\Omega = 2\omega_{o1}$), Orange - $\delta = 2.17$ and Green - $\delta = 2.43$ ($\Omega = 2\omega_{o2}$); (c) Stability boundaries in the normalized forcing control space. Blue - Parametric Instability boundary and Red - Escape boundary.

References

[1] Hollaway L. C. (2010) A Review of the Present and Future Utilization of FRP Composites in the Civil Infrastructure with Reference to their Important In-Service Properties. *Constr Build Mater* **24**:2419-2445.

[2] Mohri F., Azrar L., Potier-Ferry M. (2001) Flexural–Torsional Post-Buckling Analysis of Thin-Walled Elements with open Sections. *Thin Wall Struct* **39**:907-938