Dynamics of escape from a magnetic double-well potential

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Abstract. This work is primarily devoted to the analytical and experimental study of escape dynamics of a harmonically excited pendulum from a magnetic double-well potential. Different escape mechanisms have been explored and the escape thresholds (excitation amplitude and frequency) have been deduced by invoking action-angle variables and the method of complexification averaging.

Introduction

Escape dynamics of an excited particle from a potential well is a well-known problem in the domain of applied physics and engineering [1-3]. The escape from potential well is a transient dynamic phenomenon [1-4]. Previous studies have considered various measures to quantify escape starting with the works of Thompson [4], Virgin et al. [1, 5]. It is notable that the well-known asymptotic methods applicable for weakly nonlinear systems have a drawback since they consider the system dynamics in the near vicinity of the equilibrium points, whereas the escape dynamics manifests farther away from the equilibrium point. To this end, the recent study by Gendelman [6] invoking the canonical action-angle variables provides a good description of escape phenomena. In this study we invoke the action-angle variables and the method of complexification averaging to explore the mechanism of the escape and corresponding threshold.

The dynamical system under consideration is a harmonically excited pendulum oscillating in a magnetic field. For the autonomous system, $q = 0, \pm\pi$ are the trivial unstable equilibrium points accompanied by two centres on either side and the non-dimensional equation of motion is of the form,

$$q'' + \sin(q)\left\{1 - \frac{\alpha\beta}{(1+\alpha^2 - 2\alpha\cos(q))^{3/2}}\right\} = \varepsilon f \sin(\Omega\tau) \tag{1}$$

Where $\alpha \geq 1$ is the ratio of the height (of pivot from ground) and length of pendulum, β is the ratio of magnetic and inertia force, Ω is the non-dimensional excitation frequency and $\varepsilon \ll 1$ is the asymptotic parameter scaling the excitation amplitude f. The topology of the phase plane is shown in Figure 1 for $\beta \lesseqgtr \alpha^2 - 1$ respectively. In the presence of external harmonic excitation, the stationary pendulum at one of the centres would oscillate in its near neighbourhood. However, for a certain threshold of Ω and f, the pendulum breaches the separatrix. We consider partial (from the inner separatrix) and complete escape (from the outer separatrix) of the pendulum from the potential well. The escape trajectory (suppressing the initial transients) for one of the scenarios is shown in Figure 1 (left panel). We consider the canonical action-angle variables and the method of complexification averaging to describe the structure of escape threshold. Additionally, we have considered a table-top experiment to model this behaviour to validate the theoretical predictions.

Results and discussions

The current study considers analytical description of the mechanism and the threshold of escape of a harmonically excited pendulum from a magnetic double well potential. The escape can be either a partial or a complete escape and the corresponding thresholds have been deduced and numerically verified. A table top experiment to validate the theoretical results has been devised.

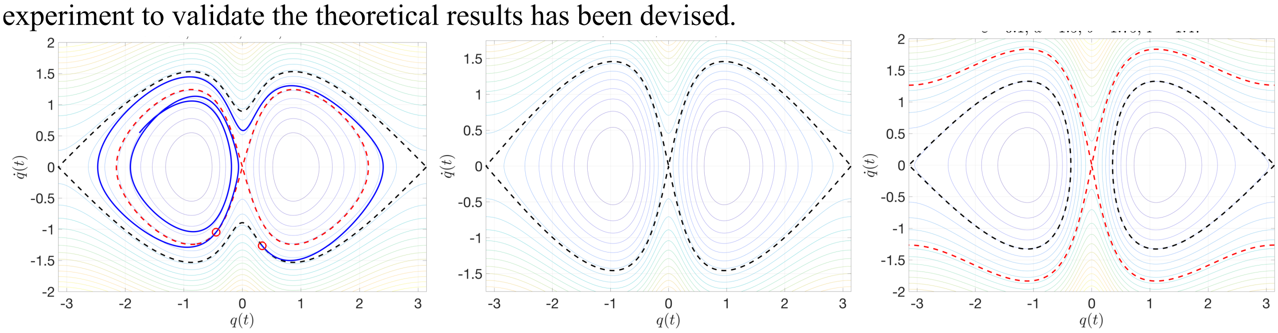

Figure 1: Phase contours (left panel) $\alpha = 1.5, \beta = 1$, (centre panel) $\alpha = 1.5, \beta = 1.25$, (right panel) $\alpha = 1.5, \beta = 1.75$

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