Analysis of Periodic and Quasi-Periodic Orbits of a Hysteretic Oscillator

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Abstract. A hysteretic relay oscillator with harmonic forcing is investigated. Periodic excitation of the system results in periodic, quasi-periodic, and unbounded behavior. The initial conditions giving rise to period-one solutions were obtained analytically. The regions of initial conditions leading to quasi-periodic and unbounded solutions were determined by numerical simulations.

Introduction

Hysteresis type nonlinearities are common for various types of systems, including mechanical, electrical and biological systems [1]. Krasnosel’skii and Rachinskii [2] investigated the bifurcations of forced periodic oscillations for systems with Preisach hysteresis. Kalmár-Nagy et al. [3] studied a single degree-of-freedom forced hysteretic system without linear restoring element. The equation of motion of the forced hysteretic system with a linear restoring element can be written as

\[ \ddot{x}(t) + x(t) + F[x(t)] = A \cos(\omega t + \phi_0), \quad A \geq 0, \omega > 0, \quad \phi_0 \in (-\pi, \pi], \]

where \(A, \omega,\) and \(\phi_0\) are the amplitude, frequency, and phase of the forcing, respectively. In this model we use a symmetric hysteresis operator

\[ F[x(t)] = \begin{cases} 
-1, & x(t) \leq -1, \\
e, & -1 < x(t) < 1, \\
1, & x(t) \geq 1.
\end{cases} \]

where \(e\) is \(-1\) or \(1\) depending on the initial conditions and the time history of the solution, i.e., whether the solution enters the hysteretic region \(-1 < x(t) < 1\) from the left or right. Without loss of generality, we specify the initial conditions as \(x(0) = -1, \dot{x}(0) = v_0 < 0, e = -1\).

Results

We derived the following expressions for the initial conditions of symmetric period-one solutions

\[ \cos(\phi_0) = \frac{A}{(\omega^2 - 1)}, \quad v_0 = \omega \tan(\phi_0) - \tan\left(\frac{\pi}{2\omega}\right). \]

Varying the forcing parameters we observed coexisting symmetric and asymmetric periodic solutions. Figure 1 illustrates the regions of initial conditions \((v_0, \phi_0)\) leading to period-one (dots), quasi-periodic (filled regions) and unbounded solutions.

![Period-one, quasi-periodic solutions](image)

(a) Forcing parameters: \(A = 2, \omega = 0.4\)
(b) Forcing parameters: \(A = 2, \omega = 0.6\)
(c) Period-one, quasi-periodic solutions

Figure 1: (a,b) Regions of period-one, quasi-periodic solutions, (c) typical trajectories

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References