Quasiperiodic solutions and stabilty chart in two degrees of freedom delayed van der Pol-Rayleigh oscillators under parametric coupling

Ilham Kirrou* and Mohamed Belhaq **

**2MGC group ENSA-Agadir, University Ibn Zohr Morocco*

***Faculty of Sciences Ain Chock, University Hassan II Casablanca, Morocco*

Abstract. Analytical approximations of the quasiperiodic (QP) response and its stability chart are investigated for a nonlinear coupled self-excited two degrees of freedom (dof) oscillator under time delay. The coupling effect is implemented through a parametrically modulated stiffness. The QP response is obtained analytically using using the second-step perturbation method. It is shown that in the presence of time delay, the QP amplitude increases substantially and for appropriate values of delay parameters, stable periodic and QP solutions may coexist in a very small region of the stability chart.

Introduction

We consider a delayed nonlinear two dof coupled self-excited system composed of two oscillators coupled by a parametrically modulated spring. Each oscillator is attached to a nonlinear stiffness and nonlinear damping. In the generalized co-ordinates, the equations of motion are given by

$$m_1\ddot{x}_1 + (-c_1 + c_2\dot{x}_1^2)\dot{x}_1 + k_1x_1 + k_2x_1^3 + (k_5 - k_0cos2\Omega t)(x_1 - x_2) = g_px_1(t-\tau) + g_v\dot{x}_1(t-\tau) \quad (1)$$

$$m_2\ddot{x}_2 + (-c_3 + c_4x_2^2)\dot{x}_2 + k_3x_2 + k_4x_2^3 - (k_5 - k_0cos2\Omega t)(x_1 - x_2) = 0 \quad (2)$$

where x_1 and x_2 are the dependent variables of positions of masses m_1, m_2, dot denotes derivative with respect to time t, c_i $(i = 1,2,3,4)$ and k_i $(k = 1,2,3,4,5)$ are damping and stiffness coefficients, respectively, k_0 and Ω are amplitude and frequency of the stiffness modulation while g_p, g_v and τ are the delay amplitudes and time delay, respectively. The system (1), (2) has been studied in the case without delay [1] in terms of synchronization and chaotic regions. The objective here is to investigate QP motion and its stability chart using perturbation method [2, 3]. The main results are presented in Fig.1. Figure 1a illustrates the frequency-response curves near the principal parametric resonance and the first natural frequency. The plots in this figure show the effect of time delay on the QP amplitude demonstrating that increasing the delay amplitude increases the amplitude of the QP response, while that of the periodic one decreases. Figure 1b presents the stability chart in the parameter plane (g_v, T) near the first natural frequency for $g_p = 0.1$, showing the existence of three regions. The *white* region where stable limit cycle exists, the *green* region in which stable QP solution occurs and the *aqua* region over which stable QP solution coexists with stable limit cycle oscillation causing bistability phenomenon. It is interesting to point out that the substantial increase of the amplitude of the QP response away from the resonance can be exploited in energy harvesting.

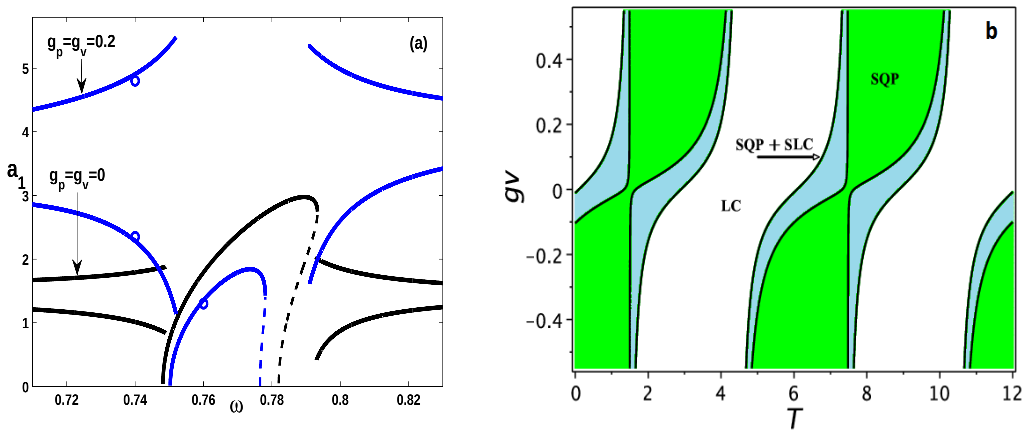

Figure 1: (a) Frequency-response curves near the first naturel frequency for $T = 3$, $g_v = g_p = 0$ for *black* line, $g_v = g_p = 0.2$ for *blue* line. (b) Stability chart in the plane (g_v, T); (LC) limit cycle, (SQP) stable QP, stable LC (SLC) for $g_p = 0.1$

References

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