## Experimental Non-Reciprocity in a Geometrically Nonlinear System Composed of Elastically-Coupled Rotators

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Abstract. We study the impulse response of a geometrically-nonlinear, elastically-linked, in-plane rotator system. A chain composed of such rotators exhibits a sonic vacuum when the pretension of the elastic links vanishes, leading to large tuneability (i.e., strong to weak nonlinearity) with small pretension. We experimentally observe broadband experimental non-reciprocity in a weakly pre-tensioned, asymmetric three-rotator model, which finds good agreement with direct numerical simulation. We then use a nonlinear normal mode analysis to uncover the observed non-reciprocity mechanism. This analysis shows that equal impulse, albeit different excitation energy, together with energy-dependent frequency/mode shapes, results in a strong mechanism for breaking reciprocity, contrary to what is found in a linear, asymmetric system.

## Introduction

Acoustic reciprocity, featuring symmetric transmission of acoustic/elastic wave between a source and receiver, can be broken by multiple methods. Compared to frequently-studied active approaches which break reciprocity via an odd-symmetry field or time-modulation, nonlinearity combined with asymmetry can break reciprocity passively while avoiding the complexity and energy consumption associated with active methods. Previous research in asymmetric nonlinear systems focused primarily on non-reciprocal behaviour in traditional rectilinear systems in which reciprocity is broken by targeted energy transfer through a nonlinear resonance mechanism [1]. In this talk, we present numerical/experimental results which demonstrate broadband non-reciprocity in a rotational system, where the nonlinearity comes from the angled elastic linkages. Further, using a nonlinear normal mode analysis, we show that the non-reciprocal behaviour is a result of excitation energy difference and energy-dependent nonlinear normal modes (NNMs), which is a fundamentally different reciprocity-breaking mechanism than observed to date.

## **Results and Discussion**

Three rotators are 3D-printed and arranged as depicted in Fig. 1a. The elastic coupling is realized by a short, lightly pre-tensioned spring and two metal rings, which enable axial extension during rotation. Different numbers of bolts and nuts mounted on each rotator form a moment of inertia asymmetry. We apply equal impulses, as required by the reciprocity theorem, to rotators at either end of the system. Responses from the other end of the chain, captured by a laser Doppler vibrometer, demonstrate non-reciprocity as depicted in Figs. 1c-d. Since the rotators directly excited by the impulse exhibit different moment of inertia, the impulses result in different imparted energy and a different initial position on the NNM frequency-energy plot (see Fig. 1e). This results in different frequencies excited in left versus right excitation, and different mode shapes. As a result, the responses to the two excitations are distinct, demonstrating non-reciprocity.



Figure 1: (a)-(b) Experimental Setup, (c)-(d) non-reciprocal response, and (e) nonlinear normal mode frequency energy plot.

## References

 Moore, K., Bunyan, J., Tawfick, S., Gendelman, O. V., Li, S., Leamy, M., and Vakakis, A. F., 2018, "Nonreciprocity in the dynamics of coupled oscillators with nonlinearity, asymmetry, and scale hierarchy," Physical Review E, 97(1), p. 012219